

Travelling Salesman Problem: THEME PARK Teacher Notes

Materials Required

Each student/pair of students will need:

- Theme Park Visitor Guide
- Theme Park Question Worksheet

Supporting Resources Available

The following resources support this session and are free to download from www.LearnAboutOR.co.uk :

- PPT,

Session Duration Guide

- For Year 10-11 / Average ability groups: Allow 60 - 90 minutes
- For Year 12-13 / High ability groups: Allow 60 minutes

Learning Objectives

- Students can convert a practical TSP into the classical TSP of finding a Hamiltonian cycle
- Students will be able to extract relevant information in order to solve a problem
- Students will understand the application of the Triangle Inequality Theorem to the TSP
- Students can determine the upper bound by applying the nearest neighbour algorithm
- Students can determine the lower bound on route lengths using minimum spanning trees
- Students are able to comprehend the complexity of complete enumeration

INTRODUCTION

Slides 2

This aim of this session is to explore the Travelling Sales Problem, see where they are used in the real world and understand how maths (and Operational Research) is used in real life.

THE P vs NP PROBLEM: The \$1 million maths problem

Time 5-10 minutes

Slides 4-7

The P versus NP problem is a major unsolved problem in computer science. The question is, if it is easy to check that a solution to a problem is correct, is it also easy to solve the problem?

P problems are easy for computers to solve – the solutions can be generated in a reasonable time compared to the problem complexity.

Contrastingly, the solutions for NP problems are very difficult to find – it could even take billions of years' worth of computation to reach a solution. However, once a solution is found, it is very easy to check. For example, finding the right arrangement of pieces to a jigsaw puzzle is difficult, but you can tell that the puzzle is complete just by looking at it.

Most people believe that $P \neq NP$. If $P = NP$ was true, it would have the potential to destabilise almost every country worldwide – economically, politically and militarily because PKC (Public Key Cryptography) would be impossible. However, it would become possible to treat cancer with extreme precision and to obtain an accurate forecast of the weather.

DISCUSSION (5 MINS): Discuss the current benefits/shortfalls and what the possible future benefits/shortfalls might be depending on whether $P = NP$ or not.

DISTRIBUTE STUDENT WORKSHEETS

BRIEF

Time 10 minutes

Slide 8

Allow students to read the Brief and answer *QUESTION 1* on their Worksheet. Encourage discussion after a few minutes. The students should recognise that they will need to consider the following information:

How long it takes to queue for and ride/tour each attraction in OCTOBER

How long it takes to queue at the park gates for entry

Allowance of 30 minutes for a lunch break

**How long it takes to walk between attractions*

After this, *DISTRIBUTE VISITOR GUIDES* – **and** allow students a further 5 minutes to find the information they need from the guide and the brief for the above.

*Explain that you will need to work together through the Travelling Salesperson Problem in order to work out the quickest route around the park.

The Travelling Salesperson, Practical vs. Classical, Completing the Network Time **SLIDES 9 - 11**

Explain that the Hamilton's Adventure Park problem is an example of a Travelling Salesperson Problem. TSP is one of the most famous and important problems in combinatorial optimisation and is an NP hard problem. It has been studied thoroughly for more than half a century.

Typically, a travelling salesperson will want to cover a number of towns by travelling the shortest distance possible.

Travelling Salesperson Problems begin to work out how to do travel the shortest distance possible by first turning the towns and roads into a network. The vertices represent the destinations (i.e. in the traditional problem these vertices would be the towns and villages that the salesman wants to visit). The edges then represent the roads between those destinations and the weights would usually represent the distances.

Triangle Inequality Theorem Time **Slides 12 - 14**

Explain that, if triangle inequality stands, it can't be more expensive (in terms of distance, time, etc.) to travel directly from one vertex to another than it is to travel via an intermediate vertex first.

Hamiltonian Cycles Time **SLIDES 15-16**

Consider the Hamiltonian cycles for this graph...

DISCUSSION: Ask the students how many Hamiltonian Cycles there are in Example 1

However, not all graphs have Hamiltonian cycles. Consider this graph...

In order to apply algorithms, any network such as this one can be replaced with a complete network of shortest distances. The shortest distance between A and C is 33 because the salesperson will have to travel via D ($12 + 21$). The shortest distance

between A and B is 35 because, again, the salesperson will have to travel via D (12 + 23). Adding direct arcs AC and AD does not change the problem but does complete the network: producing a graph with Hamiltonian cycles.

Hamilton's Adventure Park: Complete your Network SLIDES 17-20

Slide 17 needs to become Slide 18 in that the network needs to be completed

Students should complete QUESTION 2 on their Worksheet to fill in the table. ANSWERS are shown on Slide 20 to allow discussion, if necessary.

Complete Enumeration SLIDES 21-23

A Complete Enumeration algorithm for a classical problem will systematically attempt every possible solution:

- List all Hamiltonian cycles
- Find the total weight of each cycle
- Choose the cycle of least total weight (although there may be more than one such cycle)

Complete enumeration can be very expensive and sometimes even impossible for complicated problems and a T.S.P. problem doesn't have to be very big in order to become complicated...

In this problem there are only 5 vertices – five cities to visit. However, there are 4 factorial (24) different possible cycles. A T.S.P. with 10 cities/vertices will have more than 350,000 possible solutions, which begins to take time even for computers (with their current capacities) to work through in a reasonable time.

Hamilton's Adventure Park: Upper and Lower Bounds SLIDES 24-30

We have to work towards approximate solutions, such that we find the upper and lower bounds for which the weight of a minimum weight Hamiltonian cycle might be. We want the upper bound to be as small as possible and the lower bound to be as great as possible...if x and y are close together and we can identify a tour weight that is either equal to x or y or lies between x and y , that tour can be accepted as a reasonable answer – even though it may or may not be the best possible solution. In

other words, if these represent different tour weights, any lying outside of the upper and lower bounds can be dismissed.

Also commonly referred to as the “Common Sense” algorithm, the Nearest Neighbour is a greedy algorithm. This is because it takes the best option at each opportunity with no consideration for previous or future decisions. This means we have no control over the final edge and could potentially miss some very short edges.

Students should complete QUESTION 3 and QUESTION 4 on the Worksheet about Upper Bounds

A tree is a connected graph of some vertices, which contains no cycles. When all vertices of a graph are connected without any cycles, this is called a spanning tree. The MINIMUM spanning tree is the spanning tree that has the lowest weight. We could just find a minimum spanning tree by inspection, but this becomes difficult/impossible when the number of vertices increases. To find the minimum spanning tree efficiently, Prim’s Algorithm can be applied. As with the Nearest Neighbour, Prim’s is also noted to be a greedy algorithm – again, it only takes the best option available without consideration for any of the consequences of the choices made. The number of edges in a minimum spanning tree is always $(n-1)$ one less than the number of vertices Remember to make sure you don’t create a cycle.

Students should complete QUESTION 5 and QUESTION 6 on the Worksheet about Lower Bounds

Hamilton’s Adventure Park: Solution SLIDES 31-36

Students should attempt a solution...answering the final QUESTIONS on their Worksheet. Allow up to 15 minutes of independent working and engage small tables in discussion.

We now know that the optimal time taken to tour the Park will be within this range. Therefore, looking back on our tours, we are unlikely to improve our tour time of 67 minutes (1 hour 7 minutes).

The tour begins at the Park Gates and goes to the Animal Kingdom, to the Corkscrew, to the Log Flume, to The Hole, to the Roller Coaster, to the Wonder Wheel, to Carnival Land, and then back to the Park Gates.

DISCUSSION: We now know that 8 hours is the minimum required time to visit Hamilton’s Adventure Park in October. Therefore, as the coach is dropping you off at the Park Gates for 10am, it will need to collect you again at 6pm. Some of you may have decided to allocate an extra hour in the Park for toilet breaks, snacks, or extra rides. In which case, the coach will need to collect you from the Park Gates at 7pm instead.

**T.S.P. Applications; O.R.
SLIDES 37**

TSP and other network algorithms are the basis of many commercial supply chain and logistics software packages. This is important to supermarkets, haulage companies, people delivering things – e.g. DHL, FedEx, EXEL, Eddie Stobart, etc. - to save on petrol, to save on time, and to maximise profit. Companies employ people called “logistics managers” to design their route in order to make savings and increase profit. Logistics managers can earn in excess of £40,000 per annum.

**OPERATIONAL RESEARCH
Presentation Slide 38
2 minutes**

Ask the students if they have heard of operational research. Often not many people have. (Text appears on click/moving forward).

The answer on the slide can also be stated as “OR involves using maths to solve problems or make better decisions”. It is a little unspecific as an answer – that’s because OR is useful in many real-world situations!

OR is used today by many businesses – shops, airlines, architects, hospitals, local government and central government.

There are some in depth examples of OR on the following slides. Feel free to include your own.

**OR IN DETAIL - SUPERMARKETS
Presentation Slide 39
2 minutes**

Supermarkets use teams of OR professionals to solve problems and make decisions, such as understanding consumer buying patterns, deciding how many staff they should allocate to a shift and calculating the optimal quantity and delivery times of their products.

Supermarket loyalty cards, like a Tesco’s Clubcard, are a great example of OR in action. Loyalty cards let supermarkets track what their customers are buying, creating huge amounts of data for operational researchers to work with. They can use statistics to search for patterns in the data, attempting to predict how customers will behave in the future.

For example, the data might show that people buy lots of milk on a Saturday, in which case the supermarket would know to stock up on Friday evening. It might also show that lots of people shop at certain times, or on a particular day, so the store managers would know to have more staff members working at that time.

Most supermarkets also incorporate weather forecasting data, obtained from weather stations near each of their stores to optimise this further by making sure they have extra BBQ food in towns that are expecting sunny weekends.

It's easy to see what a big impact OR has on making the right decisions for supermarkets – helping them keep customers happy and make profits!

OR IN DETAIL - AIRLINES

Presentation Slide 40

2 minutes

Operational researchers at places like British Airways are involved in a lot of decision-making.

When you book a holiday, OR has been used to decide where an airline will fly to and how much they charge you for your ticket, using customer buying patterns and forecasting to predict demand.

When you arrive at the airport, OR has been used to minimise queueing times, and simulations are used to model the flow of passengers through the terminal to ensure staff members and equipment are in the right places at the right time.

When you board the plane, OR has helped choose a boarding strategy and ensure your plane leaves on time. OR is even used to forecast how many passengers are likely to cancel their holiday!

Just like supermarkets, airlines rely heavily on OR to make better, more informed decisions that result in better outcomes for their business.

OR IN DETAIL - HEALTHCARE

Presentation Slide 41

2 minutes

Some hospitals have dedicated OR teams to help with resource allocation – especially if they have multiple specialities. The OR staff allocate patients, equipment and surgical teams to operating theatres based on the urgency and specific requirements of each patient – some operations need specialist equipment and others do not and it's not very efficient to have a 'general' patient in a 'specialist' surgery.

The OR team have to set a schedule, which is made complicated by the fact that how long an operation takes can be hard to predict and an emergency patient might need immediate attention and throw off the rest of the rota!

OR researchers designed an algorithm to optimise kidney transplant surgery – imagine somebody needs a kidney transplant and their family member is willing to be a donor, but is incompatible. The algorithm identifies patients in this situation and matches them up so they can swap donors, and both patients receive the kidney that they need.

The surgery has to take place simultaneously to prevent anybody from backing out at the last minute, so the algorithm also has to take into account the nearest hospital

with enough resources (theatres and surgical teams) to carry out the transplant when matching patients.

WHEN IS OR USED?

Presentation Slide 42

2 minutes

Decision-making and problem-solving in business can be complicated and messy. It may not be clear what the main problem is, what the outcome of different actions may be or how well things are currently working, and there may be lots of different factors to consider.

For example, if things don't go well when businesses make big changes, they might upset customers, slow down production, or create a need for extra staff training. Any of these could have a negative impact on the business. OR can help to reduce the chances of this happening.

WHAT OR TECHNIQUES ARE USED?

Presentation Slide 43

2 minutes

Some commonly used OR techniques include:

Optimisation – making something more effective - depending on what variable is most important (manufacturing something quickly, or maximising profit?), optimisation will find the best use of limited resources.

Simulation – this modelling tool is fantastic when there are a lot of different ways to solve a problem as you can try lots of different solutions until you find the best one. It also allows something to be tested in a safe way, for example, organisations like the NHS have to be careful when making changes as lives could be at risk!

Forecasting – forecasting can be used to try and predict unknown factors, to help keep a business running smoothly. For example, estimating customer demand so companies know which goods to produce or forecasting the impact of rush hour traffic on a delivery route, so the driver can stay on schedule.

Also many more techniques – including algorithms!

WHERE CAN OR TAKE YOU?

Presentation Slide 44

2 minutes

So where can OR take you? Employers who recruit for O.R. analyst are large and varied, spanning across all different industries.

So this is a non-exhaustive list of businesses that use OR. These are not endorsed by the OR Society but are designed to show the variety of careers in OR.

As you can see from the slide, there are various organisations across so many industries that use OR. For example, the government is a big employer of OR analysts, with more than 25 government departments and agencies relying on OR analysts to help them find solutions to complex managements problems. Other organisations from other industries also rely on OR analysts like, EY, British Airways, IBM and the Royal Bank of Scotland just to name a few!

OR analysis will typically work with colleagues in areas such as economics, statistics, social research and science.

INTERESTED?

Presentation Slide 45

2 minutes

If you are interested in OR here are a few next steps. You can continue studying Maths at GCSE and A Level and then further on into university.

Not many universities offer OR degrees, although some offer maths and OR degrees or similar. OR is often a module in a maths or business studies degree and can be hard to find on its own.

STEM degrees (science, technology, engineering and maths) show a skill set and analytical way of thinking that is often beneficial to people working in OR and are a good alternative to an (often elusive) OR degree.

FIND OUT MORE

Presentation Slide 46

2 minutes

For more information on OR and how to get into OR, visit the OR Society website or twitter. Any questions?